Snake Models

GILSON A. GIRALDI[1], ANTONIO A. F. OLIVEIRA[1]

[1]Laboratório de Computação Gráfica
COPPE - Programa de Engenharia de Sistemas e Computação
Caixa Postal 68511, CEP 21945-970, Rio de Janeiro, RJ, Brasil
giraldi, oliveira@lcg.ufrj.br

Abstract. The original proposal of Active Contour Models, also called snakes, in the field of image segmentation and computer vision suffers from the strong sensitivity to the initial contour position and can not deal with topological changes. The T-Snake is an approach to relieve the problem of topological changes by embedding the snake model in the framework of a simplicial decomposition of the domain. T-snakes address the sensitivity to initialization by using region-based statistics to weigh an inflation force. Another approach, the Dual Active Contour Model has the specific ability of avoiding local minima by using two contours: a inner one which expands from inside the target and another one which contracts from outside it. This approach is designed to avoid local minima through the comparison of the inner and outer snake energy. In this work we present the original snake model, discuss its basic limitations and present some details of the T-snake and Dual Contour models.

1 Introduction

Active Contour Models, also called snakes, were proposed by Kass at al. [10] and since then have been successfully applied in a variety of problems in computer vision and image analysis, such as edge and subjective contours detection, motion tracking and segmentation [3].

Basically, there are two types of snake models: the implicit ones and the parametric ones.

Implicit models, such as the formulation used in [13], consist basically of embedding the snake as the zero level set of a higher dimensional function and to solve the corresponding equation of motion. Such methodologies are best suited for the recovery of objects with complex shapes and unknown topologies. However, due the higher dimensional formulation, implicit models are not as convenient as the parametric ones, for shape analysis and visualization, and for user interaction.

The parametric snake models consist basically of an elastic curve (or surface) which can dynamically conform to object shapes in response to internal forces (elastic forces) and external forces (image and constraint forces). These forces can be the result of a functional global minimization process or based on local information. Such approach is more intuitive than the implicit models. Its mathematical formulation makes easier to integrate image data, an initial estimated, desired contour properties and knowledge-based constraints, in a single extraction process [3].

However, parametric models have also their limitations. First, most of these methods can only handle topologically simple objects. The topology of the structures of interest must be known in advance since the mathematical model can not deal with topological changes without adding extra machinery[14]-[6]. Second, parametric snakes are too sensible to their initial conditions due to the nonconvexity of the energy functional and the contraction force which arises from the internal energy term [16, 9].

Several works have been done to address these limitations. The use of simulated annealing for minimization, and dynamic programming [1] have been proposed to reduce problems caused by convergence to local minima. However, the utility of such techniques is limited by performance problems [9, 4].

Levine at al. [12] used another approach by applying hierarchical filtering methods, as well as a continuation method based on a discrete scale-space representation. Basically, a scale-space scheme is first used at a coarse scale to get closer to the global energy minimum represented by the desired contour. In further steps, the optimal valley or contour is sought at increasingly finer scales.

These methods address the nonconvexity problem but not the bad effects of the internal normal force. This force is a contraction force which makes the curve collapse into a point if the external field is not strong enough.

In Cohen [5] and Gang Xu at al. [16] this problem is addressed by the addition of another internal force term to reduce the bad effects of the contraction force. In both these works the number of parameters are increased and there are some trade-offs between efficiency and performance.

Another way to remove the undesired contraction force of the snake model is to use the idea of invariance which is well known in the field of computer vision [8]. That idea has been applied for closed contours and consists in designing an internal smoothing energy, biased to ward some prior shape, which has the property of being invariant to scale, rotation and translation. In these models, the snake has no preference to expand or contract but tends to acquire a natural shape.

An example of a technique which applies invariance concepts is the Dual Active Contour [9]. This approach basically consists of one contour which expands from inside the target feature, and another one which contracts from the outside. The two contours are interlinked to provide a driving force to carry the contours out of local minima, which makes the solution less sensible to the initial position.

These works do not address the topological limitations of the traditional snake model. Basically we are interested in two topological operations: splits and merges. The proposals found in the literature to accomplish this are basically composed by two steps: first a procedure to identify the necessity of a topological operation and second a (surgical) procedure to implement it [14]-[6].

The T-snakes model is an example of such a methodology which has the advantage of being a general one. The basic idea is to embed the snake model within the framework of a simplicial domain decomposition using classical results in the field of numerical continuation methods [2]. The resulting model has the power of an implicit formulation without the need of a higher dimensional formulation.

In the T-snake model, the non-convexity of the energy functional is addressed by using region-based statistics to weigh a force used to make the snake converge to the boundary objects.

Next we present the main features of the original snake model. After, we discuss the Dual and T-snakes models.

2 Snake Models

2.1 Classical Snake Model

Snakes or Active Contour Models are a special case of the deformable model theory [3, 10]. In this work we are only interested in the 2D case although the extension to 3D can be also accomplished.

Geometrically, a snake is a parametric contour c, here assumed to be closed, embedded in a domain $D \subset \Re^2$:

$$c : [0,1] \rightarrow D \subset \Re^2; c(s) = (x(s), y(s)). \qquad (1)$$

We can define a deformable model as a space of admissible deformations (contours) Ad and a functional E to be minimized. This functional represents the energy of the model and has the form:

$$\begin{aligned} E &: Ad \rightarrow \Re; \\ E(c) &= E_1(c(s)) + E_2(c(s)); \end{aligned} \qquad (2)$$

where

$$E_1 = \int_\Omega \left(w_1 \|c'(s)\|^2 + w_2 \|c''(s)\|^2 \right) ds , \qquad (3)$$

$$E_2 = \int_\Omega P(c(s)) \, ds, \qquad (4)$$

are the internal and external energy terms, respectively. In the internal energy, the parameter w_1 (tension) gives the snake the behavior of resisting to the stretch and w_2 (rigidity) makes the snake less flexible and smoother. These parameters can be constants or dependent on s[12]. Each prime denotes a degree of differentiation with respect to the parameter s.

In the external energy E_2, P is the potential associated with the external (image) forces and is narrow related with the features we seek. For edge detection in a greyscale image a possible definition is [3]:

$$P = -\|\nabla I\|^2 , \qquad (5)$$

where I is the image intensity.

The process of minimizing the functional given in (2) can be viewed from a dynamic point of view by using the Lagrangian mechanics [7]. This leads to dynamic deformable models that unify the description of shape and motion. In these models the deformable contour is viewed as a time-varying curve

$$c(s,t) = (x(s,t), y(s,t)), \qquad (6)$$

with a mass density μ and a damping density γ.

The Lagrange equations of motion for a snake with potential energy (2) have the form[12, 15]:

$$\mu \frac{\partial^2 c}{\partial t^2} + \gamma \frac{\partial c}{\partial t} + (w_1 c'(s))' + (w_2 c''(s))'' + \nabla P(c(s)) = 0, \qquad (7)$$

where the first two terms represent the inertial and damping forces while the third and fourth terms give the forces related to the internal energy (2). The last term in equation (7) is the external force due the external potential P in (5). The equilibrium is achieved when the internal and external forces balance and the contour comes rest; which implies that:

$$\partial c / \partial t = \partial^2 c / \partial t^2 = 0. \qquad (8)$$

In order to solve the equation (7) for an initial closed contour we have to discretize the snake in space and time by using local or global representation methods each of them with trade-offs between performance and numerical efficiency [11, 12]. We have also to use a *termination condition* to stop the numerical interactions [12].

The first point to stress here is that the space Ad in (2) do not include contours with more than one connected component. So the classical snake model do not incorporate topological changes of the contour c during its evolution given by equation (7). Besides, the contraction force generated by the third and fourth terms in this equation is shape dependent and makes the stabilization of the snake too dependent of the parameters w_1 and w_2. While in theory it is possible to compute a pair of proper weights of the internal energy (3) for each point, it is very difficult in practice [16]. The following technique shows a way of addressing this problem by using an invariant internal energy.

2.2 Dual -ACM Model

In this model [9] and for the discussion that follows a snake will be considered a particle system $v_i = (x_i, y_i), i = 0, ..., N-1$ whose particles are linked by internal constraints to form a closed polygonal. In fact, such formulation can be viewed as a discrete form of the classical formulation described above.

The basic idea of the Dual ACM is to reject local minima by using two contours: one which contracts from outside the target and one which expands from inside. Such proposal makes possible to reduce the sensitivity to initialization, by enabling a comparison between the two contours energy, which is used to reject local minima. To obtain the conventional continuity and smoothness constraints, but removes the unwanted contraction force, a scale invariant internal energy function is developed.

Specifically, a *shape model* is accomplished by the following internal energy [9]:

$$E_{int} = \frac{1}{2}\sum_{i=0}^{N-1}\left(\frac{\|e_i\|}{h}\right)^2, \quad (9)$$

where (see figure 1):

$$e_i = \frac{1}{2}(v_{i-1} + v_{i+1}) - v_i + \frac{1}{2}\theta_i R(v_{i-1} - v_{i+1}), \quad (10)$$

where R is a 90^0 rotation matrix and θ_i is related to the internal angle φ_i in the vertex v_i by:

$$\theta_i = \cot\left(\frac{\varphi_i}{2}\right). \quad (11)$$

It is clear that E_{int} has a global minimum when $e_i = 0, i = 0, 1, ..., N-1$. From (10)-(11) it can be shown that this happens when

$$\varphi_i = \Theta_i = \pi(N-2)/2N, i = 0, 1, ..., N-1, \quad (12)$$

which are the internal angles of a regular polygon with vertices given by points like p_i [9]. The energy (9) can be also shown to be rotation, translation and scale invariant [9].

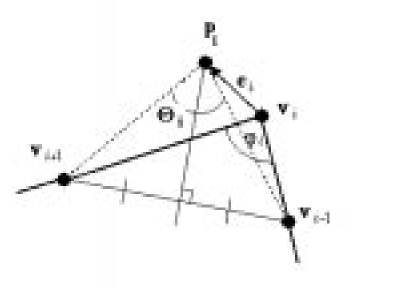

Figure 1: A Shape model for Dual approach.

The total energy of the model is given by:

$$E = \frac{\lambda}{N}E_{int}(v_i) + (1-\lambda)E_{ext}(v_i), \quad (13)$$

where E_{ext} is the discrete form of (4) and λ is a smoothing parameter which lies between 0 and 1 [9].

With the internal energy (9) the internal contraction force problems are avoided because the curve do not have the tendency to collapse but only to satisfy the constraint (12) - the curve is biased to ward a regular polygon. This fact makes also easier to establish the correspondence between the points of the two contours because the form of the snake during the evolution is limited by the energy (9). The methodology takes advantage of this correspondence by proposing the *driving force*:

$$F_{drivng} = g(t)\frac{u_i - v_i^t}{\|u_i - v_i^t\|}, \quad (14)$$

where v_i^t is the contour being processed at time t, u_i is the contour remaining at rest and $g(t)$ is the strength of the force.

The termination condition adopted in [9] is a discrete version of (8) when the model has no inertia (mass $\mu = 0$):

$$\max\left\|v_i^{t+1} - v_i^t\right\| < \delta, \quad (15)$$

where δ is a termination parameter.

The dual approach consists in making the inner and outer contours evolve according the following algorithm: The contour with the highest energy (13) is selected. If its motion remains below some termination condition then the driving force (14) is increased until it moves at a rate greater than the chosen threshold δ . When the energy begins to decrease, the added driving force is removed and the contour is allowed to come into equilibrium. The procedure is then repeated until both contours have found the same equilibrium, which is guaranteed by the direction of the driving force.

2.3 T-Snakes: A framework for topological Changes

The T-snake approach is composed basically by three components: a simplicial decomposition of the domain of interest, in our case a closed subset $D \subset \Re^2$, a particle model of the snake and a *characteristic function* defined over the simplicial which distinguish the interior from the exterior of an object O:

$$\chi : D \subset \Re^2 \rightarrow \{0, 1\} \quad (16)$$

such that: $\chi(p) = 1$ if $p \in O$ and $\chi(p) = 0$, otherwise, where p is a node of the simplicial grid.

Starting from these elements, topological operations can be reduce to find the combinatorial manifold whose dual is an one dimensional manifold that approximates the final contour as we shall see bellow. Such proposal has the advantage of importing classical results of numerical continuation [2, 15] to the context of active contours models given a general and simple methodology to deal with topological operations in the field of parametric snake models.

2.3.1 Simplicial Decomposition

Given a closed subset $D \subset \Re^n$ there are two main types of domain decomposition methods: non-simplicial and simplicial. The non-simplicial methods are based on a cell decomposition of the space (Marching Cubes is an example) and so they can not be used to represent surfaces or contours unambiguously without the use of a disambiguation scheme.

Simplicial decompositions, on the other hand, provide an unambiguous framework for the creation of local polygonal approximation of a contour or surface model. Let's see some basic definitions in this field [2].

Definition 1. A set of points $\{v_1, v_2, ..., v_{k+1}\}$ in $\Re^n$ is said to be affinely independent if the differences $v_2 - v_1, v_3 - v_1, ..., v_{k+1} - v_1$ are linearly independent vectors in $\Re^n$.

Definition 2. The convex hull with vertices given by a set of $k+1$ affinely independent points $\{v_1, v_2, ..., v_{k+1}\}$ in $\Re^n$ is called a *k-simplex*

Definition 3. Let Γ be a non-empty family of $(n+1)-simplices$ in $\Re^{n+1}$. We call Γ a triangulation of $\Re^{n+1}$ if:

a. $\sigma \in \Gamma \bigcup \sigma = \Re^{n+1}$;

b. the intersection $\sigma_1 \cap \sigma_2$ of two simplices $\sigma_1, \sigma_2 \in \Gamma$ is empty or a common face of both simplices;

c. the family Γ is locally finite i.e. any compact subset of $\Re^{n+1}$ meets only finitely many simplices $\sigma \in \Gamma$.

For $\Re^2$ a 2-simplex is a triangle and a collection $\Gamma$of triangles satisfying definition 3 above defines a *triangulation*. Following the classical nomenclature, a vertex of a triangle will be called a *node* and the collection of nodes and edges will be called the *simplicial grid* Γ_s.

Definition 4. We say that a $k-$simplex σ is a *boundary $k-$simplex* if the characteristic function (16) changes its value in σ.

In this framework, the reparameterization of a contour is done by taking the points of the contour as the intersections points of the snake with the simplicial grid and the topological changes are carried out by distinguishing the interior and exterior of the snake(s). The figure 2 shows the simplest triangulation of the plane - the Coxeter-Freudenthal triangulation- which is constructed by partitioning space using an uniform square grid and then subdividing each square into two simplices. The figure shows also a contour projected over the simplicial grid Γ_s corresponding.

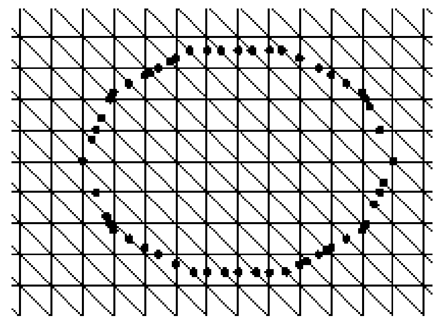

Figure 2: Simplicial grid and a contour projected.

Let's see how these ideas were incorporated in the snake model to allow topological changes. Let us take the characteristic function (16) defined over a Coxeter-Freudental triangulation of the plane and two close contours according the figure 3. The marked vertices have value 1 and the non-marked have the value zero according (16). If we trace the triangles with at least one (but not all) vertices outside the region surrounded by the contour(s) we find a two dimensional combinatorial manifold. The merge of the two curves is better represented by a curve which is the *dual* of this manifold [14]. The same would be true for more than two contours (and obviously for only one).

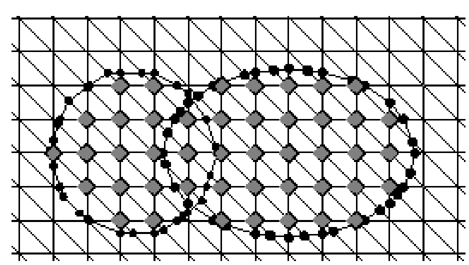

Figure 3: Two snakes colliding with the inside grid nodes and snaxels marked.

The algorithm to find the combinatorial manifold of interest and its dual is very simple and without ambiguities as we will see later (section 4).

2.3.2 Snake Model

A T-snake is a discrete form of the classical snake described in section 2.1. It is defined as a set of N particles (snaxels) whose positions

$$\{v_i = (x_i, y_i), i = 0, ..., N-1\},$$

are connected in series to form a closed contour. The particles (snaxels) may have mass m_i and damping γ_i.

Each pair of points v_i, v_{i+1} is called a *model element*. These points are linked by a spring defined by a stiffness parameter a_i and a natural length l_i . The idea is the snake resists expansion or compression only when the actual length $\|r(t)\| = \|v_{i+1} - v_i\|$ is greater or less than l_i respectively. Hence, given the deformation $e_i = \|r_i(t)\| - l_i$, we define a tension force given by:

$$\alpha_i = a_i e_i r_i(t) - a_{i-1} e_{i-1} r_{i-1}(t). \tag{17}$$

Since the set of snaxels and springs does not remain constant during its evolution, the rest length of the springs at time t is defined in terms of the length of the springs at time $t - \Delta t$. This gives the model the behavior of a viscoelastic materia [15].

In addition to the force (17) it is convenient to define a rigidity force to minimize the local curvature of the contour. In the T-snake model this force is defined by:

$$\beta_i = b_i \left(v_i - \frac{1}{2}(v_{i-1} + v_{i+1}) \right), \tag{18}$$

which attempts to minimize the distance between a point v_i and the centroid of its neighbors. The model also has a normal force

$$F_i = k n_i, \tag{19}$$

were n_i is the (outward) normal at the snaxel v_i and k is a force scale factor for which is assigned a signal (plus or minus) defined according to image features like an image threshold or statistics of the target objects [15]. This force is used to push the model towards image edges until it is opposed by external image forces.

The forces given in (17)-(19) are internal forces. The external force is defined in function of the image data, according the features we seek. One possibility is the definition (5) or a normalization of it [5].

The evolution equation for the T-snake in the discrete form is finally given by:

$$v_i^{(t+\Delta t)} = v_i^t - \frac{\Delta t}{\gamma_i}\left(\alpha_i^t + \beta_i^t + F_i^t + f_i^t\right), \tag{20}$$

where we are assuming that $\mu_i = 0$, $i = 0, 1, ...N-1$.

During the T-snake evolution some grid nodes become interior to a contour. Such nodes are called *burnt nodes*. To avoid the known problems that the evolution in the normal direction can bring (development of shocks and singularities [15]) the T-snake model incorporates an *entropy condition*: *once a node is burnt it stays burnt* [15].

T-Snakes Algorithm: The T-snake model can be summarized as follows [15]:

For each time step:

1. Compute the external and internal forces and update the snaxels positions using equation (20),

2. Compute the new snaxels(intersection between the grid and model elements),

3. Update the characteristic function (16),

4. Using the characteristic function determine the corresponding set of boundary grid triangles and compute all new model elements and snaxels

5. Determine the model elements still valid (it belongs to a boundary triangle) . Discard the other ones.

A termination condition is obtained by assigning a *temperature* to each model element based on the number of deformations steps that a simplex has remained a boundary one. A T-snake is considered to have reached its equilibrium state when the temperature of all the snaxels fall below a pre-set *freezing point*. Once a T-snake has reached its equilibrium the simplicial grid can be discarded, if desired, and the snake runs as a classical discrete snake model according to equation (20).

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